Flow around an Isolated Porous Tube with Nonuniform Wall Thickness

Kwok-On Tong* and Charles J. Knight†
Avco Everett Research Laboratory, Inc., Everett, Mass.

I. Introduction

THE scheme of introducing gas flow into an electric discharge laser (EDL) cavity through an array of porous tubes has been the subject of recent studies.[1,2] In general, a wake-like rotational flowfield evolves near the array and decays into uniform flow in the farfield due to diffusion of vorticity. Irrotational flow can only be established near the tube array if each tube has a certain ideal normal velocity distribution around its circumference.[2] As part of the general study on flow generated by porous tubes with nonuniform wall thickness, a study has been carried out for flow around a single isolated porous tube. In the case of a porous tube with symmetric wall thickness distribution that is monotonically increasing from the thin wall to thick wall section, the flow develops into a jet with its axis normal to the thin wall section. This flowfield was predicted analytically and confirmed experimentally. A summary of this study is reported in this Note.

II. Theory

The flow around an isolated porous tube with given normal velocity distribution on the tube surface is the limiting case of an array of such porous tubes with infinite tube spacing. By allowing both the tube spacing and the distance between the back wall and tube array to be much larger than tube diameter, the rotational inviscid flow model developed for the nearfield of the concentric porous tube generated flow[1] can be adopted in the present study. The numerical scheme used in Ref. 1 must be modified to account for the nonuniform normal velocity distribution around the tube circumference,[2] i.e., porous tube with varying wall thickness. This leads to an estimated boundary vorticty distribution

$$\omega_B{}^{(\nu+1)} = -\left[\frac{\partial^2\psi^{(\nu)}}{\partial r^2} + \frac{1}{r}\frac{du_r}{d\theta}\right]_B \tag{1}$$

where ω_B is the vorticity on the tube surface, ψ is the stream function, u_r is the normal velocity, the superscript ν denotes the iteration number, and the subscript B denotes variables evaluated on the tube surface. Figure 1 shows the distribution of streamlines around a single isolated porous tube with 0.95 cm o.d. and a normal velocity distribution corresponding to the ideal distribution for an array of 0.95 cm o.d. tubes with 2.54-cm tube spacing. (See Fig. 2 for the ideal normal velocity distribution.) The distribution of streamlines shows that a jet-like flow with axis normal to the thin wall section forms within a few tube diameters and is ultimately parallel for inviscid flow. This behavior is different from the case of isolated porous tube with uniform normal velocity distribution in which the flow is purely radial.

Such a jet-like flow pattern can be inferred by the following argument. Because the porous tube is operated with a large pressure drop across the porous wall in order to attain a uniform mass injection rate along the entire tube, streamlines must issue normal to the tube surface. With the velocity component in the θ direction (u_θ) equal to zero on the tube surface, the vorticity is:

$$\omega_B = \left[\frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right]_B$$

In general, $\omega_B \neq 0$ and the flow is rotational. The results of the numerical calculation discussed in the previous paragraph show that at the tube surface $|\partial u_\theta/\partial r| < |(1/r)(\partial u_r/\partial\theta)|$. Thus,

$$\omega_B \approx \left[-\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right]_B$$

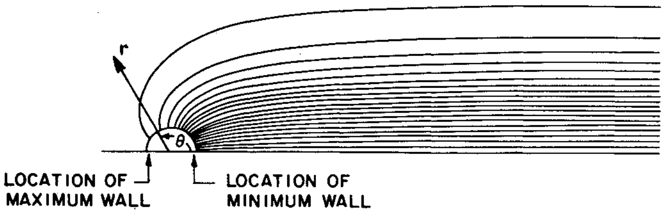

Fig. 1 Distribution of streamlines around an isolated porous tube with nonuniform wall thickness.

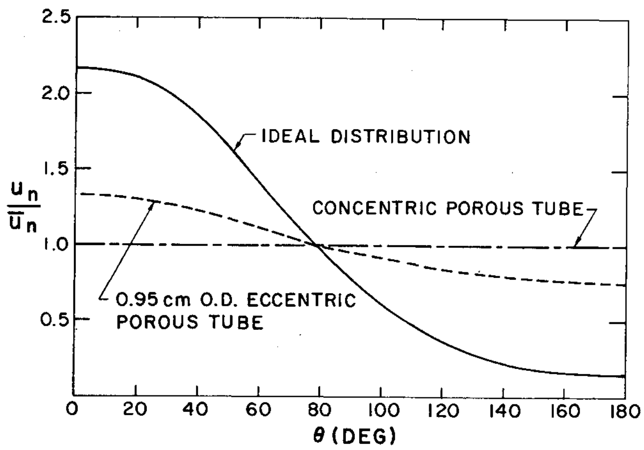

Fig. 2 Distribution of normal velocity on the surface of an eccentric porous tube.

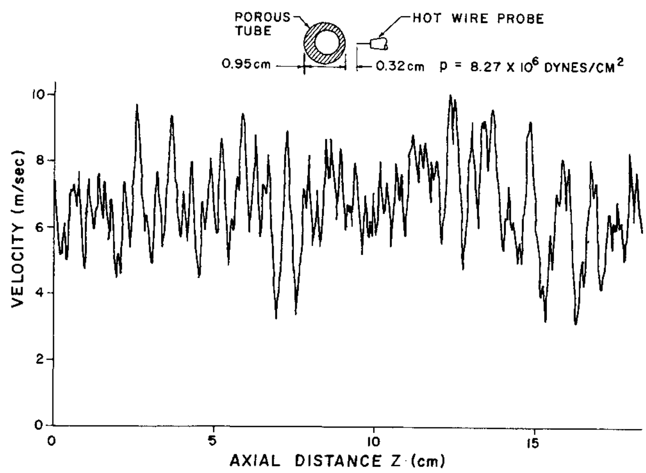

Fig. 3 Velocity distribution along the thin wall of an eccentric porous tube.

Received Aug. 29, 1978; revision received June 25, 1979.

Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Subsonic Flow.

*Senior Scientist. Member AIAA.
†Principal Research Scientist. Member AIAA.

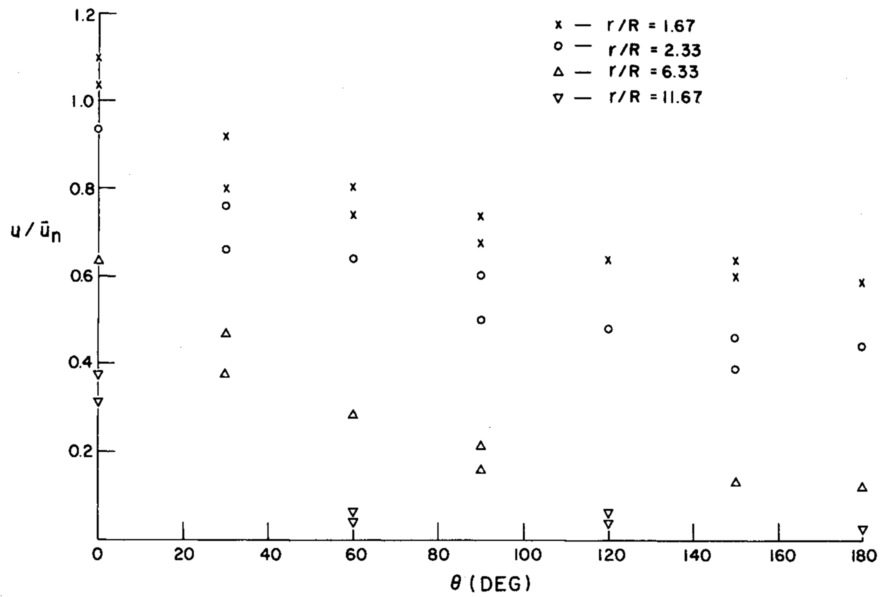

Fig. 4 Velocity distribution in the flowfield around an isolated eccentric porous tube.

The case of interest has a symmetric distribution of u_r which decreases monotonically from $\theta = 0$. With $-\pi \leq \theta \leq \pi$, $\omega < 0$ in $\theta < 0$ and $\omega > 0$ in $\theta > 0$. This distributed vorticity can be idealized as a series of vortex pairs, one above and one below the $\theta = 0$ axis, moving away from the tube surface. It is clear that as the flow expands from the surface, the vortices will tend to bend initially radial streamlines toward the thin wall section until they eventually reach an equilibrium displacement. In other words, the vortical flow from the tube surface must evolve into a jet with $\theta = 0$ as its axis.

III. Experiment

Due to practical difficulties in manufacturing the porous tubes with noncircular inner and outer contours, the porous tube for producing the ideal velocity distribution has not been developed. Instead, the eccentric porous tube studied in Ref. 2 was used in the present study. The tube used has dimensions of 0.95-cm o.d., 0.62-cm i.d., 0.11-cm thin wall thickness and 36-cm length. The porous tube has low porosity, designated by the manufacturer as having 0.5μ filtration, to attain a large pressure drop across the tube wall and thus insure uniform mass injection along the tube length. The predicted normal velocity distribution on the surface of the eccentric porous tube is shown in Fig. 2, together with the ideal velocity distribution that would lead to irrotational flow downstream of an array of 0.95-cm-o.d. tubes with 2.5-cm tube spacing. Here u_n is the normal velocity, $\bar{u}_n$ is the average velocity around the circumference of the tube, and θ is the angular location measured from the thin wall section.

Compressed nitrogen was fed in from both ends of the tube and the pressure inside the tube was maintained constant during each run. The entire tube was supported at 15 cm above the table. A constant temperature hot-wire anemometer (Thermo-Systems, Inc.) was used in conjunction with a linearizer for velocity measurement. 0.0008-cm (0.0003-in.) diam Wollaston wire was used on the hot wire probe and each probe was calibrated carefully to insure a linear net calibration curve.

Figure 3 shows the axial distribution of the velocity along $\theta = 0$ and at 0.32 cm from the tube surface. The pressure inside the porous tube was 8.27×10^6 dynes/cm^2 (120 psig). While there are variations in the velocity field, which can be attributed to small scale nonuniformities of the porous material coupled with jet coalescence effects, the axial average remains quite constant along the tube. Such axial average gives the average velocity at that particular location. Similar average velocities have also been obtained at different radial distances from the tube and angular locations from the thin wall section. The results are shown on Fig. 4, where r is the radial location measured from the center of the outer contour and R is the tube outside radius. For each r/R, the measured distribution shows that maximum velocity occurs at $\theta = 0$. The result also shows that the ratio $u_{\theta=0}/u_{\theta=180}$ increases as r/R increases. These two observations indicate that the flow around an isolated eccentric porous tube is a jet with its axis normal to the thin wall section of the porous tube.

In summary, a jet-like flowfield is theoretically predicted around an isolated porous tube with symmetric wall thickness distribution which is monotonically increasing from the thin wall section. Such behavior has been observed in the flow generated by an isolated eccentric porous tube. This particular behavior of a porous tube with non-uniform wall thickness distribution may provide a compact and convenient way of producing two-dimensional jet flow.

Acknowledgment

The authors wish to thank J. M. Avidor for his helpful suggestions. This work was supported by Naval Research under Contract No. N00014-76-C-0040.

References

[1] Avidor, J.M., Kemp, N.H., and Knight, C.J., "Experimental and Theoretical Investigation of Flow Generated by an Array of Porous Tubes," *AIAA Journal*, Vol. 14, Nov. 1976, pp. 1534-1540.

[2] Tong, K.-O., Knight, C.J., and Avidor, J.M., "Flow Generated by an Array of Eccentric Porous Tubes," *AIAA Journal*, Vol. 16, July 1978, pp. 747-749; also AIAA Paper 77-662, Albuquerque, N. Mex., June 1977.

Unsteady Flow Arising from Rotating Fluid above a Fixed Plane

W. H. H. Banks* and M. B. Zaturska*
University of Bristol, England

ONE special family of steady rotating flows which has been studied in some detail concerns the von Kármán disk problem and its generalizations: relative to a system of

Received April 24, 1979.

Index categories: Nonsteady Aerodynamics; Boundary Layers and Convective Heat Transfer—Laminar.

*Lecturer in Mathematics, School of Mathematics.